THE NEXT STEP IN METHOD*

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The topic assigned sets a task impossible for me. To foretell with even approximate accuracy the next step in the teaching procedure of mathematics is certainly beyond my powers. I shall not attempt it. But certain lines of probable development seem to be fairly well indicated from certain general trends of current educational thinking. Possibly some of you, in closer touch with the mathematical field, may from a consideration of these general tendencies be able to effect the best next step. My task this evening, however, is humbler. I shall attempt only a brief survey of what seems to me the pertinent present tendencies.

What are these general tendencies? What suggestions, if any, have they to offer to mathematics?

If I may be allowed to judge, the most widespread and imperative present tendency along methodological lines is the insistent demand that we get our students more fully "into the game." Modern education is increasingly seeing the need of treating pupils more as original centers of energy and of self-directed activity. This is not to deny an essential part in the educative process to adult guidance and control. The problem of combining both essential factors in the highest effectual degree is yet to be solved, and certainly the solution does not lie along the line of giving up control to pupils. But modern education is seeing with increasing clearness that no urge or control merely from without will ever do by or for or with students what is needed to be done. For education to be its real self, the student must somehow from within feel the urge, and from this inner urge become—increasingly as wise guidance makes possible—a real center of self-directed thought and endeavor. Otherwise there is, for me at any rate, small grounds for faith in the procedure. That this means more work from the teacher and a higher order of work is true. That it also means a higher return to all concerned is, if possible, even truer.

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Teacher and pupil alike have in the past suffered grievously from over much prescription and predigested thought. Both must in a truer sense and higher degree become, as was said above, original centers of energy and self-directed activity.

Let us examine this tendency a little more closely and, as is the wont of modern education, in the light of educational psychology. The study of the psychology of learning has made strides in our day and is now definite enough to demand our serious attention. Two lines of thought seem to have especial significance for us here: purposeful thinking and the psychological as opposed to the merely logical arrangement.

From the point of view of learning, what is the especial value of purposeful thinking and what is its lesson for us? To summarize briefly what I have elsewhere discussed at greater length, the presence of a clear and definite purpose and aim (i) supplies an inner urge for the work at hand, (ii) elicits by the psychological laws of "Readiness" and "Set" fuller pertinent thinking, (iii) supplies an end as guide to thought, (iv) adds to success a greater satisfaction, and so (v) fixes more firmly in mind and character the success-bringing thoughts. This terminology probably sounds strange to many of you and possibly even uncouth, but the facts described are known to everyone who ever found himself "absorbed" in a problem, lost perhaps to his surroundings as he grappled with some "original" in geometry or other engrossing aim. This age-old experience is sweet to all who have tasted it and fair to view in our pupils of whatever age or subject. This experience with its valuable educative results I would extend far more widely.

But this is not all. To purpose the work at hand has always been counted a valuable aid if not a necessary prerequisite to any learning. In and beyond purposing, however, there are many degrees of doing, and what the pupil learns from an experience depends greatly on the degree in which he gets into action. Consider the following as successively richer possible actions: (i) a pupil memorizes the bare words of a demonstration; (ii) a pupil memorizes the idea of a demonstration and can reproduce it in different words; (iii) a pupil makes a given demonstration his own, it becomes his thought, he can use it in a new situation; (iv) a pupil of himself demonstrates a propo-

sition that has been proposed by another; (v) a pupil of himself sees in a situation the mathematical relations dominating it and of himself solves the problem he has thus abstracted from the gross situation. Clearly a strong and definite purpose would help the pupil to success in each of these activities and would with equal certainty help fix the attendant learning the more firmly in his mind, but how different the pupil's experiences in the several instances and how markedly different the learning! Do you tell me that I am proposing the impossible, our pupils haven't the requisite ability? Possibly so, but considering these as five successive steps in an ascending scale, is it not true that the average of our pupils has in the past half century or so moved perceptibly up this scale? Did "originals" a century ago occupy so prominent a place in geometry teaching as they do now? To ask the question is to answer it. Any observed improvement, however, is not in the ability of the children concerned, if anything our pupils now are less a selected lot. No, it is our teaching that has improved, and I for one have faith that we can even yet improve. What I am proposing is that we no longer be content with stage iv as given above, that of solving problems proposed in definite terms by others, but that we move on to stage v and build up in our pupils the ability to analyze with reference to some purpose a total gross situation and to abstract therefrom the mathematical relations implicit in it and necessary to control it. Such thinking is the real thinking of life. To remain content with anything less is to be content that our pupils remain forever in tutelage, that they be henceforth underlings instead of freemen in that realm of thought we wish them to inhabit.

Possibly the discussion of the psychological vs. the logical order of presentation will make clearer the idea advanced above; for the two lines of thinking are in a true sense but correlative aspects of the same point of view. By the psychological order is meant the path the mind takes in individual discovery and experience; by the logical order is meant the scheme of arrangement wherein and whereby the mind prepares for future use the results of its experience and discovery. Suppose you undertake to solve what is to you a new and difficult "original," how many steps do you take? Imagine a faithful record made of

your whole experience of seeking. In such a complex manifold of wanderings—fruitless efforts, valid testing, finally successful effort—you have one psychological arrangement. Now contrast this with the crisp orderly arrangement in which you demonstrate the correctness of your conclusion, and you see how different the psychological and the logical can be for you at that stage of your development along that specific line.

To draw from this conception its lesson for us we must note that as teachers we are concerned not merely with the objective goals reached by pupils, but quite as truly with the actual searchings themselves. Out of a prolonged search a region of thought may be mapped. There is even ground for claiming that only out of such a wandering effort can come that organization of experience we call knowledge to mark it off from mere information. Certainly in a sense and to a degree the proposition is true. And still more, it is out of successful search that successful methods of attack must come and the courage to try. That a personal search and survey is necessary thus really to organize knowledge we cannot too much emphasize. But on the other hand an unaided search may prove unduly costly both of time and zeal. Here is the opportunity of the real teacher: What fields promise rich rewards for my group of pupil-searchers? How can I step in to save their effort from waste of time and undue discouragement and yet leave to them a real and fruitful search? The good teacher of mathematics nowadays knows, perhaps as do few others, that to have searched and found, leaves a pupil a different person from what he would be if he merely understands and accepts the results of others' search and formulation. The acceptance of this principle marks one of the definite advances made in our teaching of secondary mathematics within the past hundred years. But we may fail to realize our full possibilities here. In the ideas of the preceding paragraph this advance might mean no more than progress from stage ii or iii up to stage iv, from a stage where the child merely accepted the thought of another up to a stage where his utmost would be to solve problems formulated and propounded by another. Much more is possible.

The position I wish here to advocate is that we now advance to the next stage. Just as it is no longer sufficient that a pupil

accept Euclid's or Wentworth-Smith's demonstrations, just as personal individual work with "originals" is now seen to be necessary to give the discipline we seek, so let us be no longer content to have our pupils slavishly follow the masters' logically worked out order of thought whether in geometry or algebra, trigonometry or elsewhere. In either case the masters' thoughts may well be better than any possibly to be got out by the pupils themselves. In the one case as in the other the advocated plan may require more time, but in every case it is quite possible to gain apparent time and apparent thought at the real expense not only of zest but actually of appreciation and progress. This conception of the psychological vs. the logical order of presentation has many far-reaching implications, more than I can here even hint at, but I beg you to consider the matter. Let your thoughts play about it. On this conception we are to think of the severely logical arrangement, whether of the individual problem demonstration or of a whole topic arrangement, not at all as the initial point of attack, but as the end-outcome to be obtained only after a long and more or less wandering process. Nor need we fear: we do not make pupils logical by requiring them acceptingly to retrace our final logical formulations. Power comes from exercise of function, and usually is of slow growth. The child begins immature in practically all respects and his works manifest this all-pervasive immaturity. To shut our eyes to the fact, to pretend that it is not so, is not only to stultify ourselves, but actually to hurt our pupils. Power of logic, in the fullness that delights the learned adult, is beyond the child. He must attain it slowly. But after each experience, whether it be of his roguish four-footed playfellow or of such abstract matters as time or space, the child does in fact organize more or less adequately the results of that experience. With each such experience organized into his nervous system, he goes forth by that much a different person. The next experience in that field takes place on a correspondingly higher plane, and the youth emerges with a still more adequate organization of his previous varied successive experiences in that field. The teacher may serve as guide and helper—must as a rule so serve if undue loss is to be avoided—but the actual organization if it gets into the boy at all is put there by the boy, built up in him in and through his own efforts.

Let us now put together four thoughts taken respectively from the four last given paragraphs. First, it is in purposeful activity that we have the necessary definite aim, inner urge, full "ready" thinking, and increased satisfaction necessary—all working together—for effectual, well-organized, and abiding learning. Second, our pupils must learn to face whole situations and to abstract from each such living whole situation the mathematical relationships necessary to its control. Anything less is but abstract and unreal. Third, it is the personal search that counts, on no other basis is there real, personal, and promising organization. And fourth, the logical articulation within a whole topic and between topic and topic must itself have for each pupil a psychological history, be itself a growth, a result slowly attained from a series of successive particular organizations following particular and personal searchings. Else again we lessen the hope and promise of the boy's mathematical future.

These thoughts seems to me at least to suggest the "next step" or perhaps steps in the teaching of mathematics. If the analysis given be accepted, we must try to base our work on purposes accepted as such if not originated by the pupils. This means that we have first to begin where our pupils happen at that time to be in the matter of interests and ideas, and call into play purposes corresponding to interests actually existent. This is, of course, only a beginning. On the present as a foundation we must build up interests fruitful for continued future growing. We need not hope, such is the voice of psychology, to build any abiding and worthwhile interest except upon the foundation of a pre-existing native capacity. To begin where the child is means probably that much if not all of our first mathematics be found in other settings, perhaps of physical, perhaps of commercial or other social phenomena. Within these natural setting situations our pupils will find the needed experience in facing concrete whole situations. They will thus learn to detect mathematical relationships where perhaps on the surface none are at first apparent. Such a procedure is to be sharply distinguished from the common use of illustrations and application. Nowadays the instructor, or most often the textbook writer, first decides on the grounds o logical (demonstration) connectedness that a certain principle shall next be treated; he then hunts about for—

or better still contrives—some instance where the principle occurs among matters probably known to his prospective pupils. This he does with the idea that the principle will thereby be the more readily learned. What I am proposing is the real thing of which this common procedure is the immitation, often no better than a counterfeit.

The plan proposed is that some mathematically fruitful purpose be evoked among the matters for which the pupils—as they now are—have already strong actual or immediately potential interest. Strength of interest and consequently strength of pur pose is a first desideration; from it as we saw will come better and more effective thinking and better learning. At first the pupils will feel, and it will be true, that the mathematics used is purely subsidiary. But each instance where mathematics is thus used will be none the less the best possible application and illustration of that mathematics. If this were all and no progress beyond this were to be made, no teacher of mathematics might be necessary. This, however, is only the beginning. Successive experiences of this kind result in the accumulation of a body of mathematical knowledge and procedure. We recognize it as mathematical and give it some sort of name, but continue for a while longer to think of it solely as instrumental. As instrumental, however, we are compelled to organize it, else we can't use it well. At first we do this informally, though always definitely, but in time we become more and more systematic in our arrangement.

What all of this means by way of giving reality to the boy's mathematical thinking I need not emphasize. Learning and application go thus hand in hand. Perhaps the physics teacher will no longer sneer that he has to teach anew whatever mathematics he needs to use. As from time to time the need to control different types of situations arise new conceptions and procedures of mathematics are introduced. The new type of teacher will not fear, as occasion demands, to open up a vista of the graph of a curve, of limits and variables, or of trigonometric functions, to pupils who from our old logical demonstration order point of view are not yet ready for them. Indeed our teacher will not be restrained by any of our conventional subject divisional lines. Algebra, arithmetic, analytics, solid

geometry, trigonometry, even calculus—all is grist that comes to his mill. For he is not thinking in terms of completed logical organization, but of control of situations. And actual situations do not in life follow each other according to logical sub-divisions. Nor will he be unduly disturbed if some familiar topics by chance fail to appear. Many were better forgot. Our teacher will contrariwise ask first what ideas are needed for coping with this situation. Of any new idea needed, for such control he will ask whether the pupils can get sufficient mastery of the idea to enable them to control the situation. If yes, the teacher will then go ahead and call into play anything reasonably available, past experience, reason, intuition, authority, experiment, anything to create a working conviction and an effective grasp. Demonstration and articulation he will, if need be, leave recklessly to the future.

Does this mean haphazard teaching? Far from it, though it may well mean unpredictability. That is, the particular path of progress will not be fore-arranged or even foreseen. In other words we are basing our order of progress frankly on learning conditions and not on the bare requirements of orderly demonstration articulation. Since pupil interest and purposing are essential factors among conditions and since there are in their precise manifestations among the most unpredictable of mundane affairs, it seems probable that an element of unpredictability is a permanent factor in the proposed "next step." But this is to say nothing less than that we are proposing to introduce the human element into a part of mathematics where hitherto it has been too often lacking. The capable advanced student has always felt the warmth of this factor. Our procedure has too often denied it to the beginner.

But does unpredictability mean therefore no place for training in the teaching of mathematics? By no means. In the first place there is much room for predictability as to what will and will not happen. Humans are after all much more alike than they are different. Moreover, a proper combination of the predictable with the unpredictable is just the factor needed to give zest to all human endeavor. Our games are built on it. Mathematics teaching on the basis herein suggested will require more not less preparation, and be far and away more interesting.

Any two successive years will present such variations and such new possibilities as always to challenge the best available teaching. As the teaching of mathematics now is found, there is a persisting superstition—I could not in this presence call it more—that teachers of mathematics, as distinguished from research students in the field, either die of dry rot or write textbooks or become administrators. On the theory of this superstition teaching as teaching seems not to satisfy. The plan proposed indeed looks to the emancipation of the boy, but looks just as strongly beyond that to the emancipation of the teacher.

If we left the mathematics studied to the merely instrumental aspect as indicated above, our work would be most faulty. It is indeed highly probable—at least so I view it—that for the majority they will not progress much beyond that stage. It is fairly clear, at least their reports so indicate, that even many engineers in their thinking and appreciation never get beyond the strictly instrumental level of mathematics. Far be it from me to intimate that all must find excellence along the road chosen by those who now hear me. It cannot be demanded. It certainly is not to be expected. But there will be some who find themselves here. Many choice souls there are who find delight in the realms of "pure" mathematics. Some sooner reach the confines of their talent than do others, but as far as they go together they enjoy the like pleasures. These will soon begin to differentiate themselves from the others, from the merely instrumental crowd—shall I say from the common herd? The alert teacher will note early in some a certain inclination to dwell longer on the mathematical relations from time to time, disclosed, a certain keener wish to organize the successive results into more systematic form. For these there is born in time a different and distinct interest, a wish to pursue mathematics as mathematics and for its own sake. Obeying this psychological demand a new group must be formed to exploit the newly differentiated interest. But even here psychological methods of approach will dominate, though with gradually expanding psychological outlook larger and larger articulation will be sought, and finer and finer logical points will be attempted. I can well believe that somewhere along the line Euclid's great effort will be studied as it properly deserves. Nor will this

mean a series of lectures giving in pre-digested form what the professor has in his study thought. The students themselves—always under wise guidance—will in this special and limited field face again a concrete living whole situation. How few postulates can we get along with? What would result if we reversed this postulate? Again with trusty weapons of attack forged in many a hard-won battle—the figure, alas, is too small to hold the truth—they will analyze this—for them—living situation, and will again divide the spoil in the end, each with necessary justice keeping all he wins with much his fellows also won. And each such student will in and from such endeavors grow in a fashion denied at any rate to me when I in no mean institution studied mathematics in what was then the approved fashion of attack. Whether now I should regret for my present self the treatment I then received need not concern us. But like the youthful Hannibal of old I swore then at the altar to the cause of truth and fellow-man eternal enmity to that form of teaching which disregards the learner and the paths he needs must take to learn. For this cause I now speak these words.